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To be (finite) or not to be, that is the question "Kaluza-Klein contribution to the Higgs mass"

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Abstract

Recently five dimensional supersymmetric models with a Scherk-Schwarz supersymmetry breaking and a localized superpotential on a fixed-point have been constructed to yield a definite prediction for the Higgs mass. We examine this issue in detail and show that the finite one loop correction and the definite prediction for the Higgs mass are just a consequence of a special "Kaluza-Klein regularization" scheme.

I. INTRODUCTION

Electroweak symmetry breaking is one of the important problem in high energy physics and the clear understanding of it is still missing. Radiative electroweak symmetry breaking due to the large Yukawa coupling is one of the appealing aspects of the conventional supersymmetric models, so called MSSM (Minimal Supersymmetric Standard Model). Weak scale supersymmetry has been the most popular model beyond the Standard Model due to several attractive aspects including the stabilization of Higgs mass, the gauge coupling unification and good dark matter candidates (LSP in the MSSM), but the complete theory including the supersymmetry breaking sector and the messenger sector is quite complicated and does not look economic. As an alternative to the weak scale supersymmetry, several new ideas [1,2] considering extra dimensions got an interest for recent several years. Combining these ideas allows us to have a Scherk-Schwarz supersymmetry breaking mechanism [3–5] once the compactification radius is of order 1/TeV. Also new mechanisms for the generation of electroweak symmetry breaking has been proposed in these models. In the model [6] it has been claimed that Higgs mass can be predicted once Higgs VEV is determined from Z boson masses with the aid of the finite one loop correction of Kaluza-Klein(KK) contributions to the Higgs mass. The result is insensitive to ultraviolet(UV) physics and all the contributions of energy scales higher than the compactification scale is exponentially suppressed [7,8]. There has been a suspect [9] whether the result is just an artifact of the special regularization choice, "Kaluza-Klein regularization", or it is independent of the regularization choice. It has been shown in [9] that the result is highly sensitive to the cutoff of the Kaluza-Klein modes. Consequent papers [10–13] revealed the opposite conclusion that the UV insensitive result is robust. The result of [10] shows that the finite size resolution of the brane Yukawa coupling makes the effective Yukawa couplings of higher KK modes soften and allows us to have a UV insensitive result. In [11] it has been shown that the finiteness of the KK one loop correction is maintained in all different regularizations which keep the supersymmetry manifest. In [12,13] five dimensional mixed position-momentum space has been considered to show the ultraviolet(UV) insensitive characters of the physics. In this paper we follow the calculations given in the papers and show that the finiteness of the one loop correction is based on the infinite sum of the Kaluza-Klein states which is unphysical. Under very reasonable assumptions that the momentum cutoff should be given isotropically in five dimensional theory (p_0, p_i and $p_5 = m_{\rm KK}$), the truncated series can not be approximated by the infinite sum. It is shown that the contributions of heavier Kaluza-Klein modes are not exponentially suppressed and are equally important for given four dimensional momentum. Thus it is important to see the contributions of higher four dimensional momentum near the physical cutoff since these are the main contributions in the quadratically divergent corrections. For fixed four dimensional momentum near the cutoff, the integrand appearing in the Higgs mass one loop corrections becomes e^{-p} after summing up all the Kaluza-Klein modes, in other words infinite p_5 integration. However, this is not the right procedure since the inclusion of the Kaluza-Klein modes whose masses are heavier than the cutoff does not have any sense. The exponential suppression of the four dimensional momentum brings us the finite result even after sending our four dimensional cutoff to infinity. It is very unnatural that the conclusion can be obtained only when we summed up all the Kaluza-Klein states first. In the next section, we briefly review the basic setup.

Then one example of the series is illustrated which is directly related to the Kaluza-Klein contributions and shows clearly what gives wrong interpretations. The argument based on mixed position-momentum space is reexamined and it is shown that the supersymmetry breaking is in the bulk and exponential suppression of heavier Kaluza-Klein modes are not true. Using these, we comment on the papers claiming the UV insensitiveness that all the results are based on the infinite sum of Kaluza-Klein states. Since supersymmetry is broken even above the compactification scale (actually supersymmetry is broken at the fundamental scale in the Scherk-Schwarz mechanism). supersymmetry is not a symmetry that we should keep in choosing the proper regularization. Finally we summarize it.

II. BASIC SETUP

The constrained standard model [6] from extra dimension can be summarized as follows. [1] Consider five dimensional theory with supersymmetry whose minimal multiplet corresponds to N=2 multiplets in 4-D language. After the compactification of one extra dimension to $\frac{S_1}{Z_2 \times Z_2'}$, we obtain the Kaluza-Klein spectrum whose zero modes are the same as that of the conventional standard model. One Z_2 breaks half of the supersymmetry and the other Z_2' breaks the other supersymmetry. Among $N = 2$ vector multiplets, only vector fields can have zero modes under this orbifold compactification since others carry nonzero parity under the discrete symmetry. the quarks and leptons have zero modes after the compactification by the same reason from $N = 2$ hypermultiplets. After all if we look at the spectrum below the compactification scale $(1/R)$, it is the same as that of the standard model. In the standard model one loop correction to the Higgs mass is known to be quadratically divergent and it becomes one of the important theoretical motivations towards new theories beyond the standard model. However, in the constrained standard model with extra dimensions it is claimed that the quadratic divergences do not appear and the one loop correction is determined independently of the detailed nature of the fundamental scale physics. In this paper we examine the calculation based on physical grounds and show that the finiteness is an artifact of the Kaluza-Klein regularization, and the result of the calculation is highly sensitive to the UV physics.

III. SERIES CONVERGENCE

The question of whether the series converges is the basic one that can be asked first. Any truncation of the series gives quadratic divergences except an infinite sum reminds us divergent series. For instance, Taylor series expansion of

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots \tag{1}$$

is a convergent series when $|x| < 1$ but is not a convergent one otherwise. When we use a formula for the sum of a series, it is valid only when the series itself is convergent

[1] It is well summarized in [12], and we do not repeat the explanation in detail.

(whether absolutely or conditionally). Infinite sum has no meaning for divergent series. The remaining thing is to show that the Kaluza-Klein sum has a similar problem with the above non-convergent series.

There are two equivalent ways of calculating the one loop contributions of the Kaluza-Klein states to Higgs mass. One loop effective potential calculation is easy and we can get the mass from it by differentiating the potential by Higgs fields.

$$V_{1\mathrm{loopeff}}(\phi) = \frac{1}{2}\mathrm{tr}\sum_{k=-\infty}^{\infty}\int\frac{d^4p}{(2\pi)}\log\frac{p^2+m_{Bk}^2(\phi)}{p^2+m_{Fk}^2(\phi)}, \tag{2}$$

$$m_{Fk}(\phi) = \frac{2k}{R}+m_t(\phi), \tag{3}$$

$$m_{Bk}(\phi) = \frac{2k+1}{R}+m_t(\phi), \tag{4}$$

where m_{Bk} and m_{Fk} are masses of Kaluza-Klein states of bosons and fermions. The most important contribution is from large top Yukawa coupling and we restrict them to stop and top respectively. ϕ is a convenient expression for the Higgs field, and R is the compactification radius and k is an integer representing the quantized momentum along the fifth dimension.

Direct calculation of the one loop Higgs mass corrections involves a sum of different Kaluza-Klein contributions.

$$\begin{aligned} m_H^2 &\propto y_t^2\sum_{k=-\infty}^{\infty}\int\frac{d^4p}{(2\pi)}\left[\frac{1}{p^2+m_{Bk}^2}-\frac{1}{p^2+m_{Fk}^2}\right] \\ &\propto \sum_{k=-\infty}^{\infty}\int\frac{d^4p}{(2\pi)}a_k, \\ a_k &= \frac{m_{Fk}^2-m_{Bk}^2}{(p^2+m_{Bk}^2)(p^2+m_{Fk}^2)}. \end{aligned}$$

Since

$$a_k \sim \frac{1}{p^4}$$

at large four momentum (for $p > m_{F,B}$), at least logarithmic divergences are inevitable. For the cutoff Λ_c of the theory in which $\Lambda_c \sim N_c(\frac{1}{R})$, we naturally expect

$$m_H^2 \propto -N_c(\frac{1}{R})^2\log\Lambda_c \tag{5}$$

There are two useful expressions for the infinite series.

$$\sum_{k=-\infty}^{\infty}(-1)^k\frac{x^3}{x^2+k^2} = \frac{\pi}{2}\frac{x^2}{\sinh\pi x}, \tag{6}$$

$$\sum_{k=-\infty}^{\infty}\frac{x^3}{x^2+k^2} = \frac{\pi}{2}\frac{x^2}{\tanh\pi x}. \tag{7}$$

By substituting x to p and k to m_k with appropriate numerical coefficients, these two expressions are used to derive the finite one loop correction to the Higgs mass.

Furthermore, there are one more nice expressions that should be kept in mind for later purpose.

$$\frac{1}{2}\frac{1}{\sinh \pi x} = \sum_{n=-\infty}^{\infty} e^{-(2n+1)\pi x} \qquad (x > 0) \tag{8}$$

The above expressions are the key ingredients in deriving the finiteness of the one loop correction to the Higgs mass. The above series is definitely a convergent series and passes the first test. Is it enough that the series is a convergent one? To have an intuition of what is going on, let us look at one example.

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots \tag{9}$$

converges for any value of x. However, unless $x \ll 1$, first several terms of the series show divergent behavior. For concreteness, let us take $x = 100$. The left-handed side gives extremely tiny value $e^{-100} \sim 10^{-50}$, which is nearly zero. On the other hand, the right-handed one is

$$1 - 100 + 5000 - \cdots \tag{10}$$

which looks like a divergent series with an amplifying magnitude and alternating sign. The general profile is given in fig. 1 and fig. 2. Furthermore, there are two ways of grouping the series. First, summing the series with grouping two terms with (+,−) order shows that

$$\begin{aligned} e^{-x} = &(1-x) + \frac{x^2}{2!}(1-\frac{x}{3}) + \cdots + \frac{x^{98}}{98!}(1-\frac{x}{99}) \\ &+\frac{x^{100}}{100!}(1-\frac{x}{101}) + \cdots \end{aligned} \tag{11}$$

and the first 50 pairs give huge negative values for the series with $x = 100$. However, the result is very different for a different pairing,

$$\begin{aligned} e^{-x} = &1 - x(1-\frac{x}{2}) - \frac{x^3}{3!}(1-\frac{x}{4}) + \cdots - \frac{x^{99}}{99!}(1-\frac{x}{100}) \\ &-\frac{x^{101}}{101!}(1-\frac{x}{102}) + \cdots \end{aligned} \tag{12}$$

which shows that the first 50 pairs give extremely huge positive values with $x = 100$. Generally the maximum of the envelop is at $N = x$ ($N = 100$ in the example with $x = 100$ and $N = 5$ for $x = 5$ as in fig. 3).

FIGURES

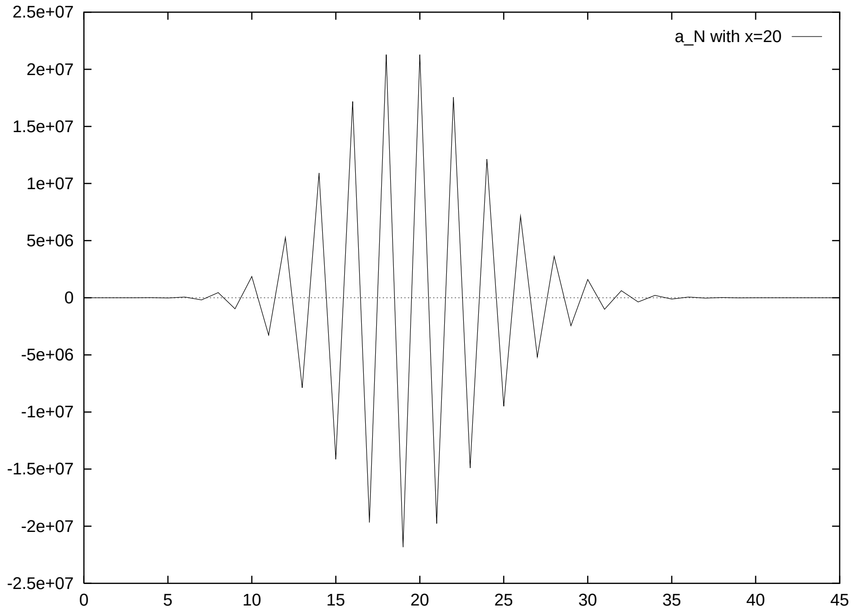

FIG. 1. Divergent and convergent behavior of the truncated series for $x = 20$. $a_N = \sum_{k=0}^{N}(-1)^k \frac{x^k}{k!}$ and for $N = 0$ to $N = 45$.

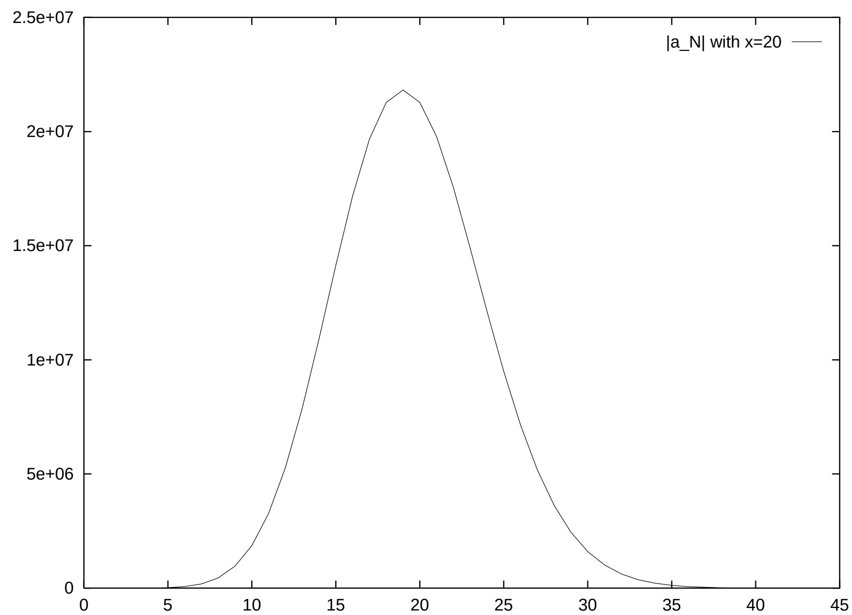

FIG. 2. Absolute value shows the magnitude of the truncated series for $x = 20$. For all regions $|a_N|$ is much larger than e^{-x}.

Fig. 3 shows at which values of N the truncated series has a similar size with the exact

series (e^{-x}). For $x = 5$, it is near $N \sim 15$. Generally only beyond $N = ex$, the truncated series starts to show the convergent feature.

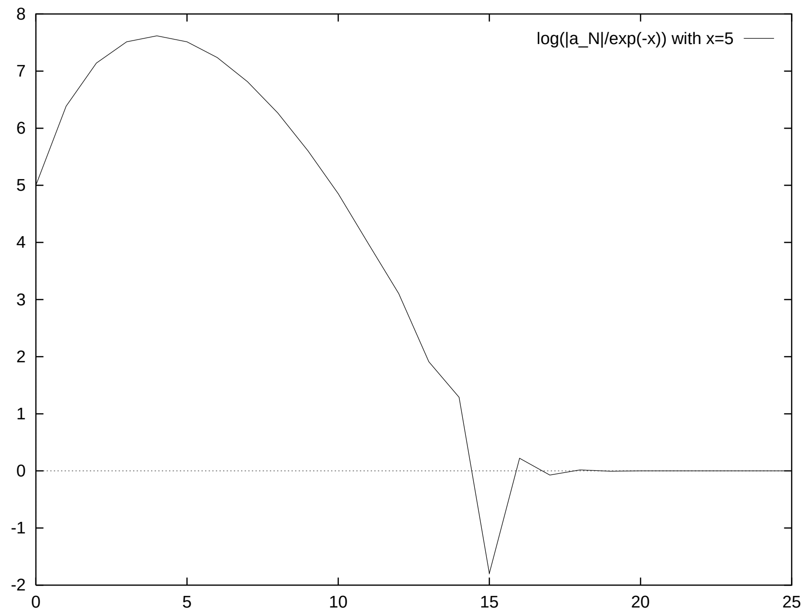

FIG. 3. Logarithm of the ratio between a_N and e^{-x} for $x = 5$. For $N > 5$, the ratio approaches to 1 (log approaches to 0), and only for $N > 15$, the ratio is close enough to 1 (log of the ratio is close enough to zero).

We can get a very important lesson from this example. Even convergent series can show a strange behavior for the truncation with finite sum. If the truncation happens in the region that we can not see the convergent feature, the approximation of replacing the sum over the finite modes to the infinite sum is not valid anymore. Therefore to know which point is the appropriate truncation capturing the underlying physics is crucial in understanding the justification of the Kaluza-Klein regularization scheme. Since we are dealing with effective theories, there is a physical cutoff scale in the theory beyond which our equipment (for example, perturbative quantum field theory) is inappropriate. The cutoff is at or below the fundamental scale in the five dimension which can be at most the four dimensional Planck scale. Before reaching this fundamental scale, we usually encounter the breakdown of the perturbation for gauge couplings and Yukawa couplings. It is usually 10 or 100 times higher than the compactification scale due to the rapid power running behavior in higher dimensions. Given the cutoff in the theory, it is not physical to take into account all of the Kaluza-Klein states which are heavier than the cutoff scale. From the five dimensional theory point of view, Kaluza-Klein mass is nothing but the momentum along the fifth dimension. The cutoff M_* apply to the momentum p_0, p_i and p_5 equally in five dimensions. This means that the momentum cutoff in the integral should be comparable to the truncation of the Kaluza-Klein mass terms which is p_5. If we accept this physical cutoff regularization, the truncation point is located near the maximum of the envelop of the series. In the example, the point is $k = x$ which shows a very divergent character. The cutoff of the four momentum

and Kaluza-Klein mass are highly correlated in the five dimensional point of view, and the result based on the infinite sum of Kaluza-Klein states is not justified for any value of the four momentum cutoff. For all values of the cutoff Λ_c, the series shows very unpredictable result which is highly sensitive to the UV physics. The sign can not be fixed and this fact reflects the quadratic divergent features directly. The quadratic divergence means incalculability of the one loop, and the usual Λ_c^2 contribution from the momentum cutoff in analytically continued Euclidean space shows merely the order of the corrections with undetermined sign.

A loop hole should be filled up in the above statements. It is the validity of the momentum cutoff regularization. Usually good regularization should preserve the symmetry that the original theory has since we can not capture the physics correctly otherwise. We are dealing with scalars and fermions, there is no problem related to the gauge invariance. In [10–13] the momentum cutoff regularization has been taken as inappropriate one due to the fact that it does not keep the supersymmetry that the original theory possesses. We show that there is no supersymmetry remained after the Scherk-Schwarz compactification. This will be discussed in detail in the next section.

IV. COUNTER TERM

In the paper [6,7] the inability of writing down the counter term for the Higgs mass has been used as one of the supporting evidence for the finite result. If there is no way to write down a counter term at tree level, this fact can be used as a proof that the one loop calculation should be finite. It has been claimed that the counter term can not be written due to supersymmetry and discrete R parities. At one point $y = 0$, one supersymmetry is unbroken and the usual μ term is forbidden due to the parity. (H_1 is even and H_2 is odd.) At other point $y = \frac{\pi}{2}R$, the other supersymmetry survives and also the μ term to different chiral multiplet (H_1' and H_2') is not allowed due to the different R parity. There is a full N=2 supersymmetry in the bulk and the mass terms for the hypermultiplets are forbidden also by R parity. Combining these facts seems not to allow a mass term for the Higgs scalar, $m_H^2 h^2$).

However, this argument is not precise and is based on several misunderstandings of the setup. To understand how the supersymmetry is broken in the extra dimension, it is helpful to look at the wave functions of the component fields belonging to the same supermultiplets. Fig. 4 shows one of the examples.

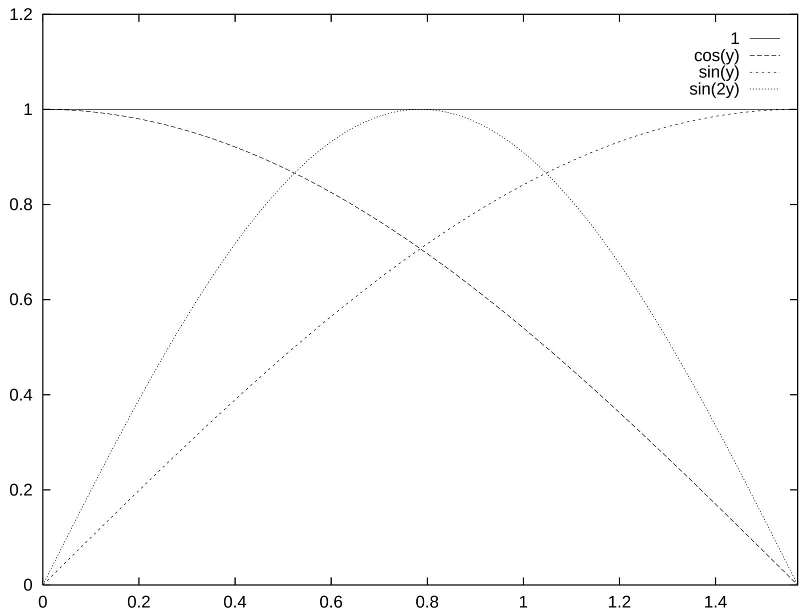

FIG. 4. Wave functions of first four modes in one $N = 2$ multiplet along the fifth dimension. Two of them form $N = 1$ multiplet at $y = 0$ (1 and $\cos(y)$), and other two form $N' = 1$ multiplet at $y = \pi/2$ ($\sin(y)$ and $\sin(2y)$). ($R = 1$ unit) Away from $y = 0$ and $y = \pi/2$, two wave functions are different and supersymmetry can not be maintained anymore.

The restoration of one supersymmetry at the boundary is seen from the agreement of two wave functions forming $N = 1$ supermultiplets. Just slightly away from the fixed point, we can see the difference of two wave function profiles belonging to the same supermultiplets and this can be interpreted as the breaking of the supersymmetry in the bulk. Table 1 shows a qualitative difference between the naive understanding of the supersymmetry breaking and the actual situation.

		$y = 0$ (one fixed point)	bulk	$y = \pi R/2$ (the other fixed point)
SUSY (claimed in [6,7])	$N = 1$	broken	unbroken	unbroken
	$N' = 1$	unbroken	unbroken	broken
SUSY in real situation	$N = 1$	broken	broken	unbroken
	$N' = 1$	unbroken	broken	broken

If supersymmetry is broken and is mediated by heavy particles, then it might be true that the contributions beyond the compactification scale ($\frac{1}{R}$) is exponentially suppressed and there may be a chance to get an answer which depends only crucially on the physics at the compactification scale. However, in the model considered here, the supersymmetry is broken in the bulk and we start to feel it just away from the brane. Therefore, the argument that forbids the counter term for the scalar Higgs mass is not valid and $m_H^2 h^2$ is allowed in the bulk since the supersymmetry is broken already. This indicates that the Scherk-Schwarz SUSY breaking does not have a soft nature as you might expect from the Kaluza-Klein regularization procedure.

V. SOFTNESS/HARDNESS OF SCHERK-SCHWARZ SUSY BREAKING

Scherk-Schwarz SUSY breaking uses different boundary conditions for bosons and fermions along the compact extra dimension. This condition has been imposed from the starting point and the SUSY breaking scale is the fundamental scale at which the theory is defined, even though the supersymmetry breaking itself can be small enough by tuning the phase small or making the compactification radius larger. It is very different from the usual supersymmetry breaking mechanisms in which supersymmetry is broken spontaneously at intermediate energy scales which are relatively low compared to the fundamental scale. For example, in gravity mediated one, SUSY breaking scale is of order 10^{13} GeV, and in gauge mediation, it can vary from a few tens TeV to intermediate scales. Since the Scherk-Schwarz mechanism gives a hard breaking of supersymmetry, there is no need to use a special regularization incorporating supersymmetry in it.

The best way to look at the softness/hardness of Scherk-Schwarz SUSY breaking is to investigate the wave functions of each KK modes in the bulk. The direct five dimensional calculation might be a good way to look at this problem. However, the indirect way of using KK mode wave functions also shows the physics we want. Since the wave functions belonging to the same $N = 1$ supermultiplets are different away from the fixed point, supersymmetry is broken except the fixed point. This result is well summarized in the table 1. This observation is very crucial in understanding the one loop correction to the Higgs mass. It is argued that the contribution from high momentum is exponentially suppressed because the fluctuations happen in a small region near the fixed point and can not feel the supersymmetry breaking effects located far apart. This argument does not hold once supersymmetry is broken just away from the fixed point. Even at high energies far above the compactification scale, the fields involved in the loop feel the supersymmetry breaking through the difference of the wave functions in the bulk and these yield quadratic divergences in the one loop calculation of the Higgs mass. Furthermore, the loop integral is over the four momentum and this does not reflect the physics discussed above. The momentum we are interested in is the momentum along the fifth dimension p_5, in other words Kaluza-Klein mass. If we carefully look at the contributions of heavy Kaluza-Klein modes ($m_k \gg 1/R$), it is not exponentially suppressed. This observation also shows that the Scherk-Schwarz breaking of supersymmetry does not have a soft nature. This is in accord with the observation of the bulk supersymmetry breaking through the wave function difference between the fields belonging to the same supermultiplet.

VI. OTHER REGULARIZATIONS

There exists a different viewpoint to the same problem. If the starting theory is the ultimate theory and we know everything from the start, then perhaps we should not truncate KK modes at any arbitrary finite order. However, we know that 5-dimensional theory also has a fundamental scale at which the gravity couples strongly, and this prevents us from consideration of full Kaluza-Klein modes. Thus we started from a theory whose limit is clear. There is an upper bound for the cutoff which is below the Planck scale. In this situation, to consider KK modes whose mass is larger than the Planck scale is not the right thing that we should do. If the series is convergent, then the answer would not depend

strongly on the truncation of KK modes and we can say that our physical observable is really UV insensitive. However, any truncation of infinite KK mode sum yields extremely strange result and it shows that the result has a strong UV dependence.

The vanishing of tree level Higgs mass is now nothing but the special choice of the counter terms which may not be the proper one. Generic one loop correction overwhelms the tree level one, and the choice of zero mass at the tree level is unnatural. In all places of the extra dimensions they feel the SUSY breaking. The SUSY breaking does not reduce the degree of divergences for the scalar mass. Therefore, Scherk-Schwarz SUSY breaking is the hard breaking and quadratic divergences reappear. In this paper we have shown that the Kaluza-Klein regularization is just one of special mathematical regularization and other regularization gives $y_t^2/16\pi^2 N_c(\frac{1}{R})^2$.

Now let us look at the papers supported the UV insensitiveness. First, consider Gaussian spreading of the Yukawa couplings [10]. Finite thickness $l_s = 1/\Lambda_s$ of the brane gives a field theoretic resolution of the apparent UV divergences. For the branes with Gaussian distributions (or equally for Yukawa couplings with Gaussian spreading),

$$f(y; l_s) = \frac{1}{\sqrt{2\pi} l_s} e^{-\frac{y^2}{2l_s^2}},$$

the effective couplings of the nth KK modes are

$$y_t^n = y_t \exp\{-\frac{1}{2}(\frac{M^n}{\Lambda_s})^2\}.$$

The above equation clearly shows that the suppression effects due to the finite thickness of the brane appear at Λ_s which is usually regarded as the fundamental scale. This fact just tells us that the Kaluza-Klein modes heavier than Λ_s do not give an important contribution to the Higgs mass correction. Therefore, this method can be used as one of alternating procedure to the finite N truncation of Kaluza-Klein modes with momentum cutoff. Unfortunately, this classical regularization still gives us misleading answer if we sum up all the Kaluza-Klein modes. The Poisson resummation formula is used in [10] and it is essential in deriving the finite answer. There is no physical interpretation for new summation index after the Poisson resummation. The reason why they got the finite answer is the same as the one of infinitely thin brane limit, that the anisotropic treatment of the four momentum and the Kaluza-Klein mass. We can apply the cutoff regularization in this thick brane setup with the cutoff $\Lambda_c \sim \Lambda_s$ and get the same divergent feature as long as the KK modes are truncated at the same scale with the four dimensional momentum. If the brane is thicker than the size of the fundamental scale, $\Lambda_s \ll \Lambda_c$, then the answer may give UV insensitive result. However, the Kaluza-Klein spectrum and wave functions should be obtained with considering these nontrivial brane configuration and the simple Fourier decomposition can not be a solution any longer. Certainly this is not a setup that was originally proposed.

In [12] (and also in [7]) the one loop effective potential takes the expression that is completely dominated by the compactification scale. This reminds us of the equation 8.

$$\frac{1}{2}\frac{1}{\sinh \pi x} = \sum_{n=-\infty}^{\infty} e^{-(2n+1)\pi x} \qquad (x > 0) \tag{13}$$

In the one loop correction to the Higgs mass, after the sum of the infinite Kaluza-Klein modes, we obtain the inverse of the hyperbolic sin function, and it can be expanded as a power series of e^{-2x}. However, at this moment there is no physical meaning to this summation index n. One thing that should be clearly distinguished is the physical meaning of the four dimensional momentum and the Kaluza-Klein modes. In the calculations using "Kaluza-Klein regularization", the suppressed contributions are not for the KK modes but for the four momentum. Therefore, this result is nothing to do with the explanation that the suppression is due to the separation of supersymmetry breaking away from the brane since the relevant object feeling the fifth dimension is the momentum along the fifth dimension, i.e., the Kaluza-Klein modes. Furthermore, as already shown in the previous section, supersymmetry is broken even in the bulk. Therefore, the UV softness of the setup is an artifact of the infinite Kaluza-Klein mode sum.

VII. SUMMARY AND DISCUSSION

We have shown that Scherk-Schwarz mechanism for the supersymmetry breaking has a hard breaking nature such that radiative corrections to the Higgs mass is quadratically divergent. This is clearly seen from the contributions of each modes with the isotropic momentum cutoff regularization. The isotropic momentum cutoff regularization, applying the same cutoff scale to the four momentum and the Kaluza-Klein mass, is the physical one since these are components of the five momentum to which we should apply the cutoff. The "Kaluza-Klein regularization" is an extremely anisotropic regularization in this point of view. The supersymmetry is broken everywhere except the fixed point in the Scherk-Schwarz breaking, and the exponential suppression of higher momentum (KK modes) does not happen. The apparent exponential suppression of higher four momentum is nothing but a sequel of the infinite KK modes sum and is not a reflection of the physics behind it. We have shown that in the physical cutoff procedure, the KK modes near the cutoff is not exponentially suppressed and this entangles the four momentum and the result remains very sensitive to the Kaluza-Klein modes near the cutoff. As a result, quadratic divergences for the four momentum do not disappear.

In most calculations people use the infinite sum of KK modes. However, it is doubtful since large log term is unavoidable if we have different mass scales in a wide range especially in the dimensional regularization scheme. Thus it is more appropriate to use an effective theory after integrating out the heavy particles. The best way to get the correction is the approach based on integrating out and matching. In this way we can reproduce the well known result of power law running of gauge couplings and Yukawa couplings which reflect properties of higher dimensional theories. However, on the contrary to the usual theory in which the mass spectrums are given below the fundamental scale and we can integrate out heavy fields step by step, the Kaluza-Klein decomposition always put their spectrums across the fundamental scale of the higher dimensional setup. It remains an open question whether we can find the correct procedure dealing with the Kaluza-Klein modes without knowing the quantum theory of gravity in higher dimensions. There are interesting works done recently related to this [14–18] utilizing the transverse lattice

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REFERENCES

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B**429, 263 (1998). I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B**436, 257 (1998).

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999); Phys. Rev. Lett. 83, 4690 (1999).

[3] I. Antodiadis, S. Dimopoulos, A. Pomarol and M. Quiros, Nucl. Phys. **B**544, 503 (1999).

[4] A. Delgado, A. Pomarol and M. Quiros, Phys. Rev. **D**60, 095008 (1999).

[5] A. Delgado and M. Quiros, Nucl. Phys. **B**559, 235 (1999).

[6] R. Barbieri, L. J. Hall and Y. Nomura, `hep-ph/0011311`.

[7] N. Arkani-Hamed, L. Hall, Y. Nomura, D. Smith and N. Weiner, `hep-ph/0102090`.

[8] L. Hall and Y. Nomura, `hep-ph/0103125`.

[9] D. M. Ghilencca and H. P. Nilles, `hep-ph/0103151`.

[10] A. Delgado, G. V. Gersdorff, P. John and M. Quiros, `hep-ph/0104112`.

[11] R. Contino and L. Pilo, `hep-ph/0104130`.

[12] Y. Nomura, `hep-ph/0105113`.

[13] N. Weiner, `hep-ph/0106012`.

[14] N. Arkani-Hamed, A. G. Cohen and H. Georgi, `hep-th/0104005`.

[15] C. T. Hill, S. Pokorski and J. Wang, `hep-th/0104035`.

[16] H.-C. Cheng, C. T. Hill, S. Pokorski and J. Wang, `hep-th/0104179`.

[17] N. Arkani-Hamed, A. G. Cohen and H. Georgi, `hep-th/0105239`.

[18] H.-C. Cheng, C. T. Hill, and J. Wang, `hep-th/0105323`.